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Pressure Vessel Design for High or Cryogenic Temperatures

Vessel design in accord with the ASME Code requires consideration of the effect of temperature on the maximum stress, but the Code offers no help in determining this effect. This article presents a group of charts with which to find both pressure and thermal stresses.

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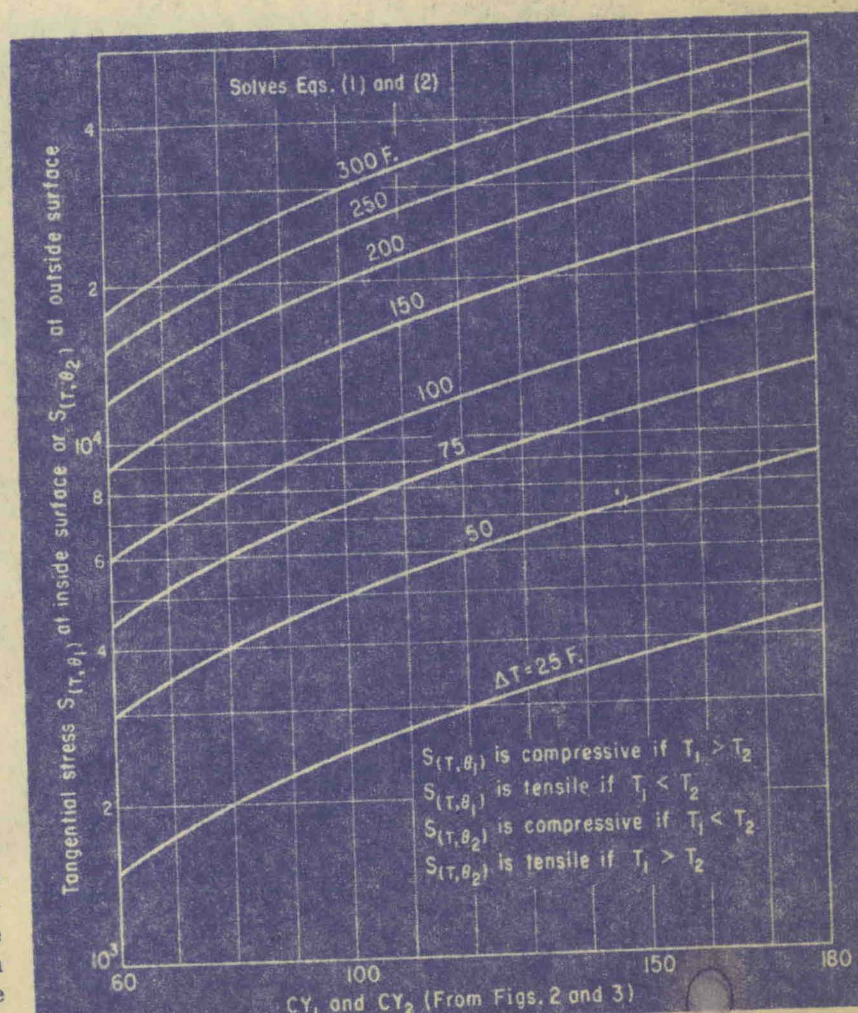
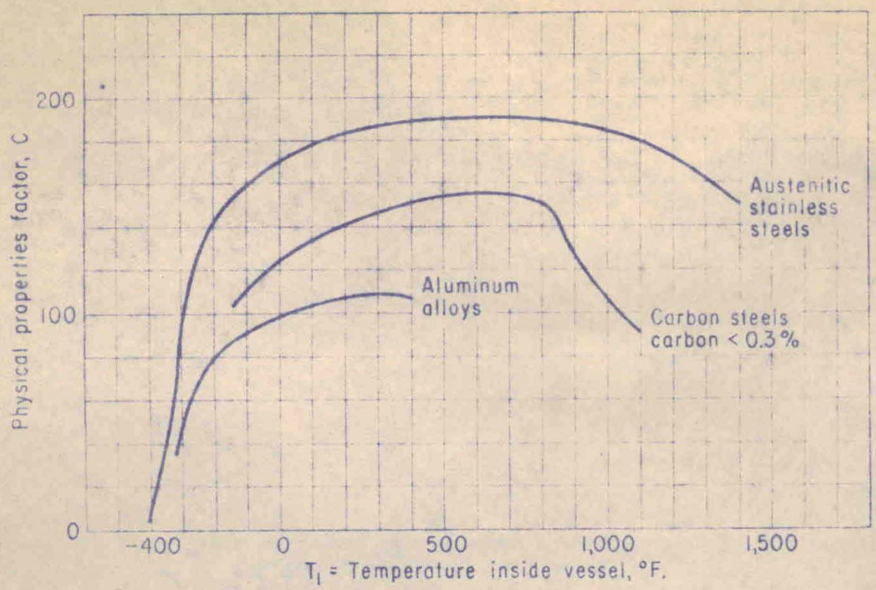


Steady-state thermal stresses under isothermal heat flow on inner and outer surfaces of a thick-walled shell; solves Eqs. (1) and (2).—Fig. 1

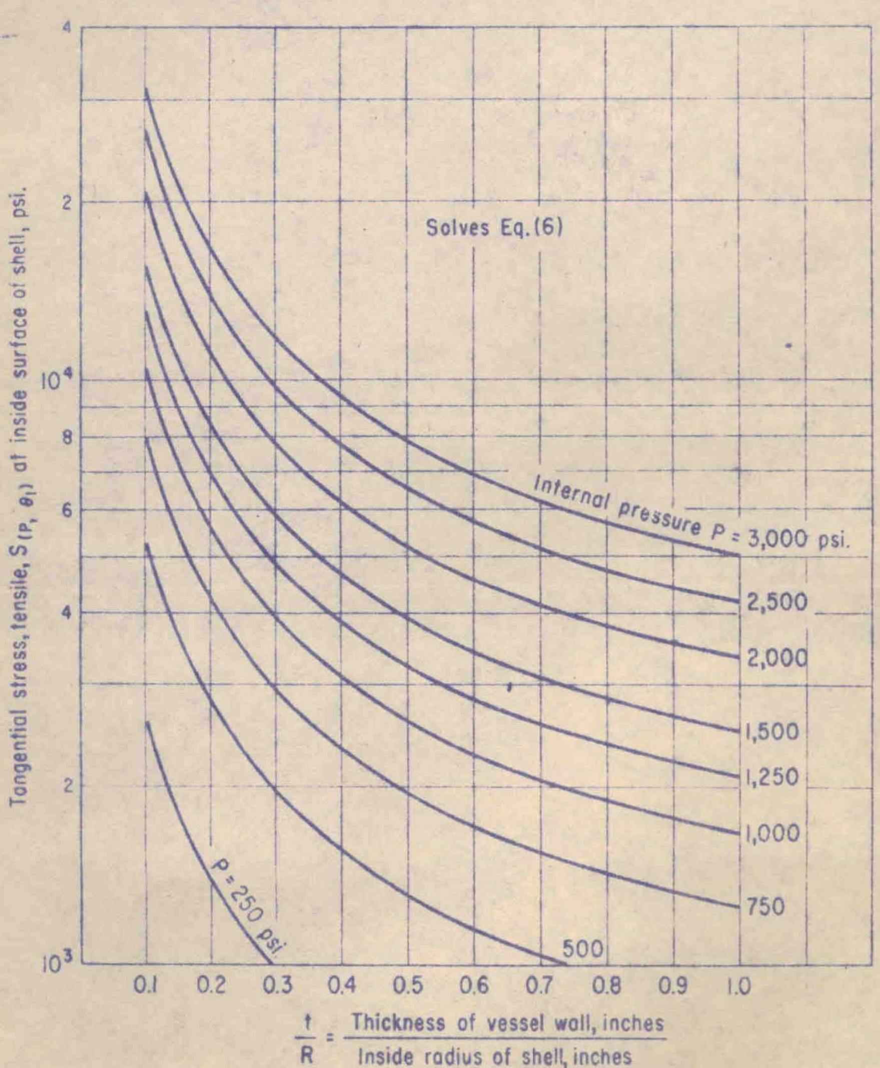
With the wider use of high-pressure processes operating at extremes of temperature, either at the high or at the low end of the temperature spectrum, it has become more necessary for the design engineer to pay attention to the combined effects of both pressure and thermal stresses.

Many vessels are designed to conform to the ASME Unfired Pressure Vessel Code[1] and compliance with Par. UG-22, Item 7, is mandatory as concerns the thermal stress loading. However, although the paragraph stipulates that vessel design loadings shall include the effect of temperature gradients on maximum stress in the vessel, the Code does not give any specific guidance toward this evaluation.

Consequently, chemical engineers



Temperature-dependent physical-property factor C for stainless and carbon steels and aluminum alloys; used in Eqs. (1) and (2).—Fig 2



Tangential stresses in the inner surface of a thick-walled vessel subjected to internal pressure; solves Eq. (6)—**Fig. 4**

are prone to underestimate the significance of such stresses in vessel design, instead leaning heavily on the factor of safety employed. It is possible for neglect of such thermal stresses to lead to conditions similar to the over-temperature hazards in pressure vessels discussed by Rossheim *et al.*[9]

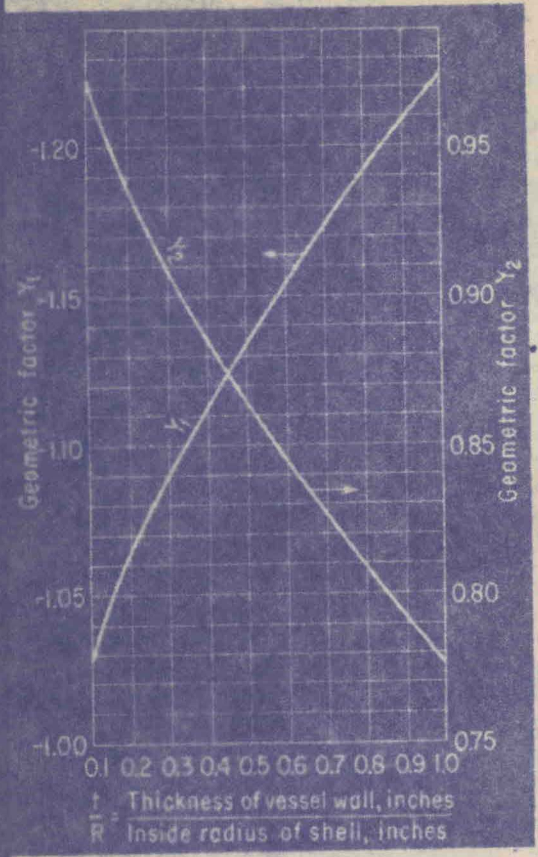
Code Limits Allowable Stresses

In addition to requiring that the vessel design be based on the simultaneous application of the pressure and temperature expected in normal operation, the ASME Unfired Pressure Vessel Code limits the maximum allowable stress in either tension or compression to the values permitted for the materials as listed in Subsection C. It is obvious that this stress limit must be based on the simultaneous application of both the temperature and the pressure stresses existing in the vessel to comply with present Code requirements. Compliance insures that the material stresses will remain within the elastic limits of the material of construction.

Although it is possible to design vessels beyond the elastic limits, for conditions beyond the scope of the Code, this requires careful consideration of many factors. Among these are transient temperature conditions, cyclic loadings, creep or plastic flow, ductility exhaustion as a result of thermal fatigue, metallurgical phase changes under sustained exposures to temperature, as well as oxidative or corrosive attacks from the vessel's contents which change the physical properties of the material of construction of the vessel.

Charts Find Thermal Stresses

In order to simplify the calculations of the combined pressure and thermal stresses as required for compliance with Code requirements, the author has developed a series of charts which are presented here along with examples dealing with the more usual conditions of stress distributions in cylindrical vessels operating under steady-state temperatures.



Dimensionless geometric factors Y_1 and Y_2 for cylindrical shells of varying t/R ratio; used in Eqs. (1) and (2).—Fig. 3

Temperature stresses become significantly large mainly in thick-walled vessels where there can be a temperature difference between the inner and outer surfaces of the vessel shell. Therefore, the charts are set up to aid in designing vessels of wall thickness anywhere in the thickness-to-radius range t/R of 0.1 to 1.0.

Calculation of the thermal stresses due to a temperature gradient across the vessel wall is based on the simplifying assumption that there is steady heat flow through the wall. For thick-walled vessels, this leads to a logarithmic temperature gradient, as discussed in Refs. 3, 4 and 5. The thermal stress equations, as derived through the theory of elasticity for regions other than the vessel ends, may be conveniently simplified and expressed in the form of single terms of the geometric parameter t/R to make their use more tractable.

The following relationships give the values of the thermal stresses for the inside and outside surfaces of the shell only, since the governing design stress can usually be determined by these values, as Examples 1 and 2 will show:

$$S_{(T,\ \theta 1)} = CY_1\Delta T \qquad (1)$$

$$S_{(T,\ \theta 2)} = CY_2\Delta T \qquad (2)$$

$$S_{(T,\ z1)} = S_{(T,\ \theta 1)} \qquad (3)$$

$$S_{(T,\ z2)} = S_{(T,\ \theta 2)} \qquad (4)$$

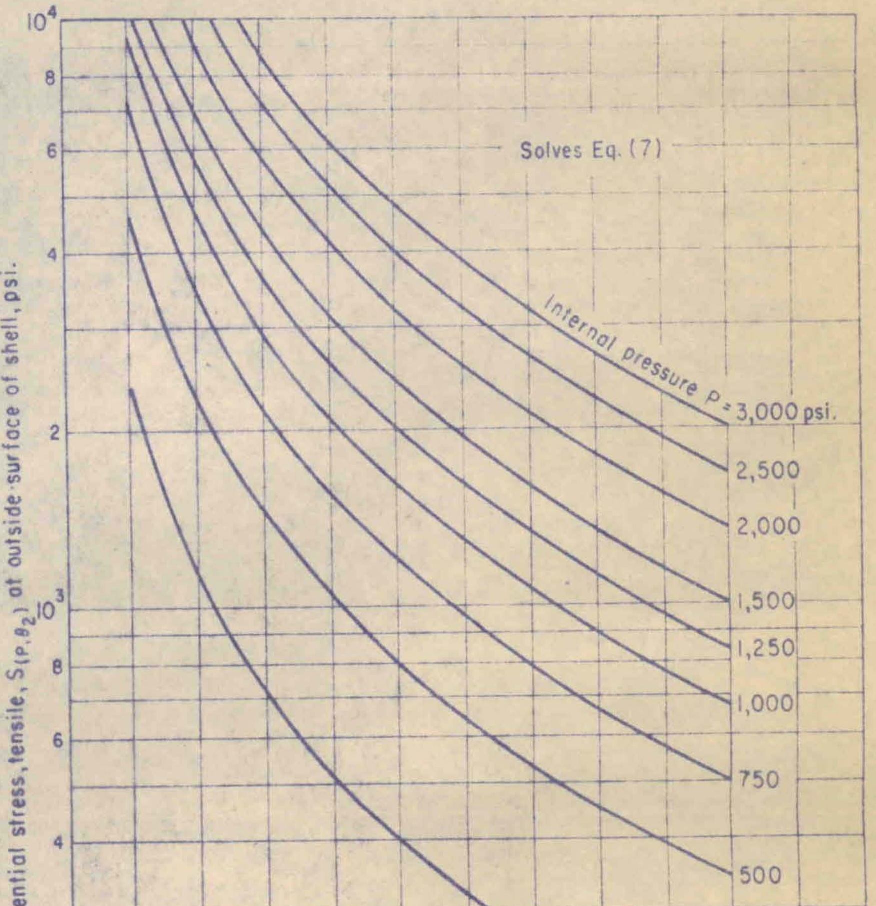
$$S_{(T,\ R1)} = S_{(T,\ R2)} = 0 \qquad (5)$$

where $C = \alpha E/[2\ (1 - \mu)]$ and

$$Y_1 = \left[\frac{1}{\ln\ (1+t/R)} - \frac{2}{1 - \dfrac{1}{(1+t/R)^2}}\right]$$

$$Y_2 = \left[\frac{1}{\ln\ (1+t/R)} - \frac{2}{(1+t/R)^2 - 1}\right]$$

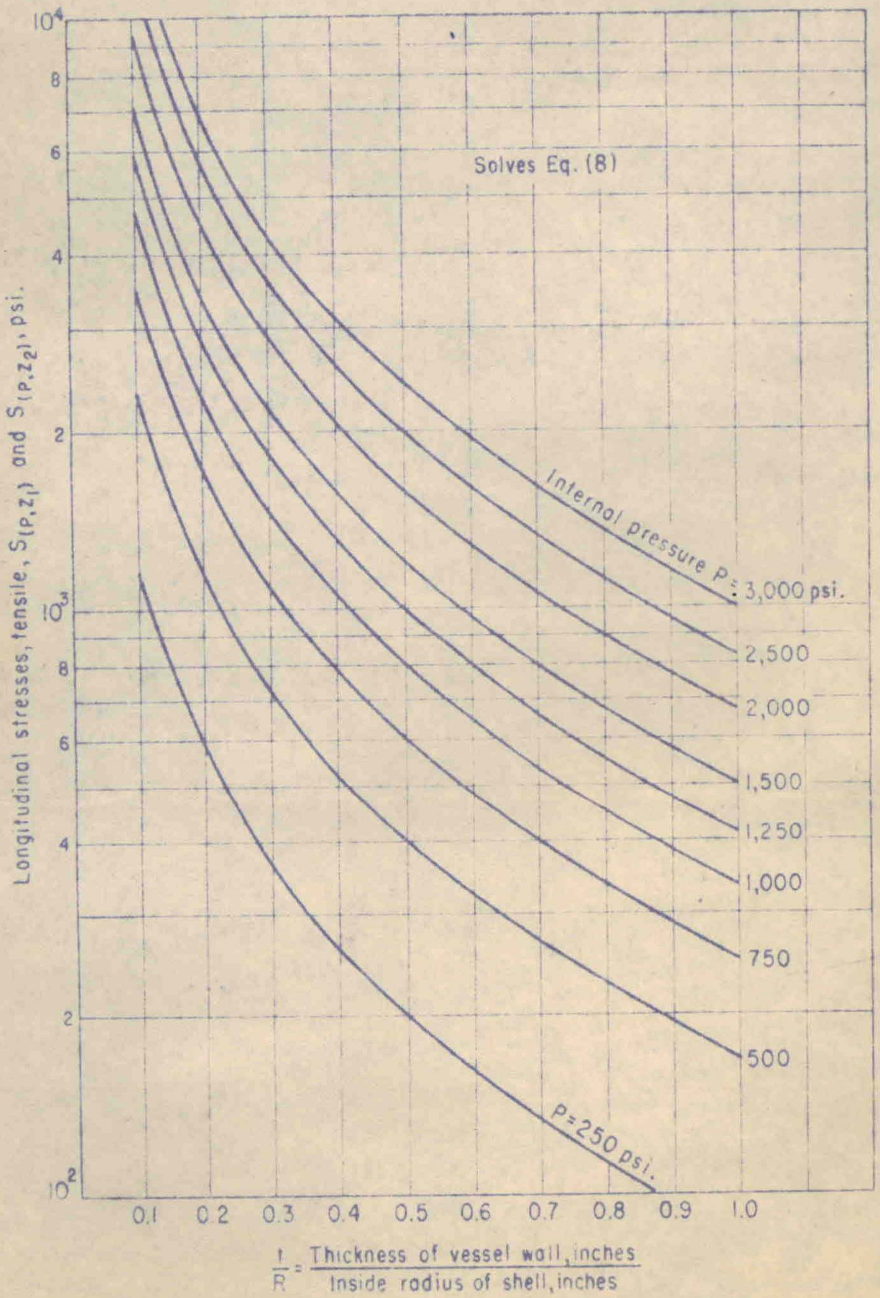
Other terms are as given in the table of nomenclature on page 136.



Tangential stresses in the outer surface of a thick-walled vessel subjected to internal pressure; solves Eq. (7).—Fig. 5

The importance of determining an accurate temperature difference ΔT at operating conditions will be obvious from these relationships which bring out its direct effect on thermal stress. Insulation and ambient conditions play a controlling role on vessel design. In addition, the temperature dependent properties of the modulus of elasticity and the coefficient of expansion assume significance at the extremes of operating temperatures as will be evident from Fig. 2. Data on the thermal properties of the metals most commonly used in vessel construction will be found in Refs. 6 through 11.

Since the thermal stress equations are based on the temperature difference existing at the operating temperature, the average of the instantaneous coefficients of expansion at the inside and outside temperatures of the shell gives an accurate measure of the shell strains. The coefficients of expansion vary only slightly within the range of ΔT, so the inside temperature of the shell may be considered, within a practical degree of accuracy, as suitable for determining the instantaneous coefficient of thermal expansion. Poisson's ratio remains essentially constant at both low and high temperatures as indicated by the constancy of the relationships between the modulus of elasticity in tension and that in torsional rigidity.



Longitudinal shell stresses, assumed constant throughout the thickness, in thick-walled vessels subjected to internal pressure; solves Eq. (8).—Fig. 6

Find Pressure Stresses Too

The similar relationships for the stresses in the vessel wall due to internal pressure alone can also be expressed in terms of the same geometric parameter t/R used for thermal expansion. Here the formulas are based on the Lamé equations as indicated in Refs. 3, 4 or 5:

$$S_{(P,\,\theta 1)} = P\left[1+\frac{2}{(1+t/R)^2-1}\right] \quad (6)$$

$$S_{(P,\,\theta 2)} = P\left[\frac{2}{(1+t/R)^2-1}\right] \quad (7)$$

$$S_{(P,\,Z1)} = S_{(P,\,Z2)} = P\left[\frac{1}{(1+t/R)^2-1}\right] \quad (8)$$

$$S_{(P,\,R1)} = -P \quad (9)$$

$$S_{(P,\,R2)} = 0 \quad (10)$$

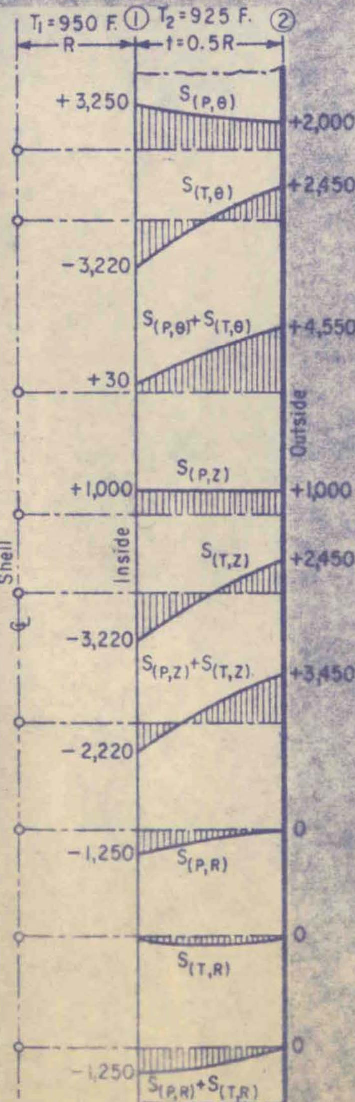
The tangential pressure stress given by Eq. (6) is equivalent to the design formula of Par. UA-2 in Appendix I of the ASME Unfired Pressure Vessel Code. The Code also allows the use of the modified membrane shell formula of Par. UG-27c when thickness of the shell does not exceed $0.385SE$. This alternate equation may be used in the calculations for the tangential pressure stress without significantly changing the results.

Eqs. (1) and (2) are solved by Fig. 1. The physical properties factor C and the dimensionless shape factors Y_1 and Y_2 are found from Figs. 2 and 3. Eqs. (6), (7) and (8) are solved directly in terms of t/R and the internal pressure P by means of Figs. 4, 5 and 6. The use of these equations is illustrated in Examples 1 and 2. Here, having computed independently the

Example 1: High Pressure and High Temperature

Conditions—Internal pressure $P = 1{,}250$ psi. Inside temperature $T_1 = 950$ F., with ΔT at 25 F. Shell material is carbon steel, SA-201 Gr. A. Joint efficiency = 1. Maximum allowable stress (tensile or compressive) = 4,500 psi. at 950 F. (Code table UCS-23). Note (*) marks governing design stress.

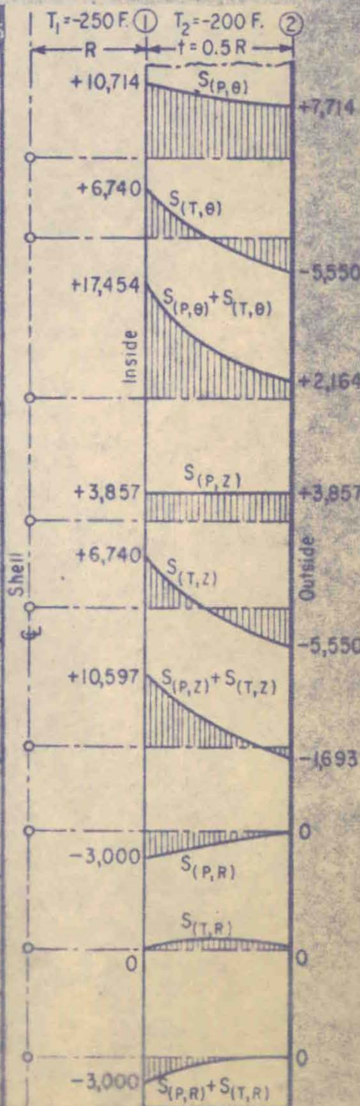
	Stress & Eq.	Fig. No.	$t/R = 0.33$	$t/R = 0.5$
Tangential Shell Stresses	Due to internal P			
	$S_{(P,\theta_1)}$ Eq. (6)	4	+4,460	+3,250
	$S_{(P,\theta_2)}$ Eq. (7)	5	+3,210	+2,000
	Due to ΔT			
	$S_{(T,\theta_1)}$ Eq. (1)	1, 2, 3	−3,120	−3,220
	$S_{(T,\theta_2)}$ Eq. (2)	1, 2, 3	+2,560	+2,450
	Stress Summation			
	$S_{(P,\theta_1)} + S_{(T,\theta_1)}$	4+1	+1,340	+30
	$S_{(P,\theta_2)} + S_{(T,\theta_2)}$	5+1	+5,770*	+4,450*
Longitudinal Shell Stresses	Due to internal P			
	$S_{(P,z_1)}$ Eq. (8)	6	+1,605	+1,000
	$S_{(P,z_2)}$ Eq. (8)	6	+1,605	+1,000
	Due to ΔT			
	$S_{(T,z_1)}$ Eq. (3)	1, 2, 3	−3,120	−3,220
	$S_{(T,z_2)}$ Eq. (4)	1, 2, 3	+2,560	+2,450
	Stress Summation			
	$S_{(P,z_1)} + S_{(T,z_1)}$	6+1	−2,515	−2,220
	$S_{(P,z_2)} + S_{(T,z_2)}$	6+1	+4,165	+3,450
Radial Shell Stresses	Due to internal P			
	$S_{(P,R_1)} = -P$ Eq. (9)		−1,250	−1,250
	$S_{(P,R_2)} = 0$ Eq. (10)		0	0
	Due to ΔT			
	$S_{(T,R_1)} = 0$ Eq. (5)		0	0
	$S_{(T,R_2)} = 0$ Eq. (5)		0	0
	Stress Summation			
	$S_{(P,R_1)} + S_{(T,R_1)}$		−1,250	−1,250
	$S_{(P,R_2)} + S_{(T,R_2)}$		0	0



Example 2: High Pressure and Low Temperature

Conditions—Internal pressure $P = 3{,}000$ psi. Inside temperature $T_1 = -250$ F., with $\Delta T = -50$ F. Shell material is stainless steel, SA-240 Type 316 L. Joint efficiency = 1. Maximum allowable stress (tensile or compressive) = 17,500 psi. (Code limits stress to value at −20 F.). Note (*) marks governing design stress.

	Stress & Eq.	Fig. No.	$t/R = 0.33$	$t/R = 0.5$
Tangential Shell Stresses	Due to internal P			
	$S_{(P,\theta_1)}$ Eq. (6)	4	+17,300	+10,714
	$S_{(P,\theta_2)}$ Eq. (7)	5	+14,300	+ 7,714
	Due to ΔT			
	$S_{(T,\theta_1)}$ Eq. (1)	1, 2, 3	+ 6,500	+ 6,740
	$S_{(T,\theta_2)}$ Eq. (2)	1, 2, 3	− 5,780	− 5,550
	Stress Summation			
	$S_{(P,\theta_1)} + S_{(T,\theta_1)}$	4+1	+23,800*	+17,454*
	$S_{(P,\theta_2)} + S_{(T,\theta_2)}$	5+1	+ 8,520	+ 2,164
Longitudinal Shell Stresses	Due to internal P			
	$S_{(P,z_1)}$ Eq. (8)	6	+ 7,150	+ 3,857
	$S_{(P,z_2)}$ Eq. (8)	6	+ 7,150	+ 3,857
	Due to ΔT			
	$S_{(T,z_1)}$ Eq. (3)	1, 2, 3	+ 6,500	+ 6,740
	$S_{(T,z_2)}$ Eq. (4)	1, 2, 3	− 5,780	− 5,550
	Stress Summation			
	$S_{(P,z_1)} + S_{(T,z_1)}$	6+1	+13,650	+10,597
	$S_{(P,z_2)} + S_{(T,z_2)}$	6+1	+ 1,370	− 1,693
Radial Shell Stresses	Due to internal P			
	$S_{(P,R_1)} = -P$ Eq. (9)		− 3,000	− 3,000
	$S_{(P,R_2)} = 0$ Eq. (10)		0	0
	Due to ΔT			
	$S_{(T,R_1)} = 0$ Eq. (5)		0	0
	$S_{(T,R_2)} = 0$ Eq. (5)		0	0
	Stress Summation			
	$S_{(P,R_1)} + S_{(T,R_1)}$		− 3,000	− 3,000
	$S_{(P,R_2)} + S_{(T,R_2)}$		0	0



stresses due to pressure and to temperature, we sum them algebraically to give the resultant tensile or compressive stress distribution in the vessel wall. It will be noted in both examples that the problem is solved twice, once for $t/R = 0.33$ and again for $t/R = 0.50$. In each case it will be observed that a thickness ratio of $t/R = 0.33$ could handle the job without exceeding the allowable maximum stress if internal pressure alone where producing wall stresses. The addition of temperature stress, however, in each case makes it necessary to go to a t/R of 0.5 to maintain the combined stresses below the allowable limit permitted by the Code.

At the right of each example is a graph of the stress pattern through the vessel wall, with tensile stresses shown as positive and compressive stresses negative.

Now Try Examples

Example 1 has been selected to show that the summation of the tangential stresses at the outside of the shell governs the design. In Example 2 the flow of heat is from the outside to the inside of the shell and ΔT is considered as negative, thereby reversing the sign of the thermal stresses. In this case, the summation of the stresses at the inside of the shell dictates the design. Although longitudinal and radial stresses normally do not govern design, their values are included to complete the stress pattern.

Such stress patterns apply only to the steady-state conditions of the vessel. Transient conditions, as the vessel reaches a steady-state level, warrant special attention. An indication of the stress pattern may be obtained by selecting several levels of pressure and temperature other than the final steady-state level, and employing the corresponding allowable stresses and thermal properties. These indications of stress pattern should not be construed as a substitute for a more rigorous analysis under transient temperature conditions, but it may serve to guide the engineer toward further investigations.

Although the calculations of Examples 1 and 2 have indicated the stress values for a given temperature difference ΔT, it is advisable to recheck the computation of the temperature gradient based on the final thickness selected for the shell.

In applications where the allowable working stresses are based on creep limitations of the materials used for vessel construction, some relief of the thermal stresses may be expected to occur under operating conditions. However, the recognition of relaxation of this sort has not as yet become a part of the Code, although such plastic flows may be within the elastic limits.

Nomenclature

C — **Temperature-dependent physical properties factor $= \alpha E/[2(1-\mu)]$; see Fig. 2.**
E — **Modulus of elasticity, psi. at T_1.**
P — **Internal pressure, psi.**
R — **Inside radius of vessel shell, in.**
$S_{(P,\theta 1)}$ — **Tangential pressure stress at inside surface of shell, psi.**
$S_{(P,\theta 2)}$ — **Tangential pressure stress at outside surface of shell, psi.**
$S_{(P,z1)}$ — **Longitudinal pressure stress at inside surface of shell, psi.**
$S_{(P,z2)}$ — **Longitudinal pressure stress at outside surface of shell, psi.**
$S_{(P,R1)}$ — **Radial pressure stress at inside surface of shell, psi.**
$S_{(P,R2)}$ — **Radial pressure stress at outside surface of shell, psi.**
$S_{(T,\theta 1)}$ — **Tangential thermal stress at inside surface of shell, psi.**
$S_{(T,\theta 2)}$ — **Tangential thermal stress at outside surface of shell, psi.**
$S_{(T,z1)}$ — **Longitudinal thermal stress at inside surface of shell, psi.**
$S_{(T,z2)}$ — **Longitudinal thermal stress at outside surface of shell, psi.**
$S_{(T,R1)}$ — **Radial thermal stress at inside surface of shell, psi.**
$S_{(T,R2)}$ — **Radial thermal stress at outside surface of shell, psi.**
t — **Shell thickness, in.**
T_1 — **Inside surface temperature of shell, °F.**
T_2 — **Outside surface temperature of shell, °F.**
ΔT — **Temperature difference between inner and outer surfaces of shell; positive when heat flow is outward, negative when heat flow is inward.**
Y_1 — **Dimensionless geometric shape factor, from Fig. 3.**
Y_2 — **Dimensionless geometric shape factor, from Fig. 3.**
α — **Instantaneous coefficient of expansion, in./in./°F. at T_1.**
μ — **Poisson's ratio, 0.3 for steels, 0.33 for Al alloys.**

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